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LETTER TO THE EDITOR

DISCUSSION OF THE PAPER "ON INERTIAL COUPLING IN DYNAMIC EQUATIONS OF COMPONENTS IN A MIXTURE" (B. T. CHAO, W. T. SHA and S. L. SOO, *Int. J. Multiphase Flow* 4, 219–223, 1978).

The authors rightly point out that there is some degree of inertial coupling in most two-phase flows and that these effects can suffice to make the characteristics real. However, they give the misleading impression that this coupling can be represented by a single unique expression that is valid for all flow regimes. I believe this to be incorrect.

Inertial coupling is predominantly determined by the kinematic constraints imposed on relative motion by the geometry of the interface between the phases. If relative motion contains a strong three-dimensional component or induces local variations in the axial velocity there will be significant inertial coupling. Thus, for example, two fluids flowing in parallel axial streamlines side by side are not coupled at all inertially while a suspension of thin discs oriented perpendicular to the axis of motion is so strongly coupled to the fluid that relative motion is hard to establish without the intervention of an external force. These processes do not resemble the collisions between molecules in a mixture of gases (the authors' only physical basis for [2]) and lead to inertial coupling that does not necessarily involve the phases symmetrically; for example, there is no parallel inside the body to flow around a solid sphere.

Inertial coupling enters the conservation equations because of non-one-dimensional components of the motion that influence the "averaging" of the densities and fluxes of momentum and energy. The average velocities are essentially defined by the continuity equations. The momentum flux may contain terms other than $u_i \rho_i^2$ $(i = 1, 2)$ that need to be evaluated from a knowledge of the flow regime. For example, in certain cases (e.g. drop-annular flow, flows of flocculated suspensions) it may be reasonable to assume that a certain fraction C of phase 2 is entrained in phase 1 and moves at speed u_1. Evaluation of the combined momentum flux reveals that it is given, in the notation of Chao, Sha & Soo, by $\rho_1 u_1^2 + \rho_2 u_2^2 + \rho_2(u_1 - u_2)^2(C/1 - C)$.

Similarly, in another case (e.g. a porous material and a fluid flowing together, flows of certain foams) the motion of the fluid phase 1 relative to phase 2 may be constrained to occur at an angle β to the axis of motion. The momentum flux of the combined phases is $\rho_1 u_1^2 + \rho_2 u_2^2$; however, the kinetic energy flux is $\frac{1}{2}\rho_1 u_1^3 + \frac{1}{2}\rho_2 u_2^3 + \frac{1}{2}\rho_1 u_1 \tan^2 \beta (u_1 - u_2)^2$. When these equations are combined to yield separate "equations of motion of each phase" a term proportional to $\partial(u_1 - u_2)/\partial x$, that could be given the significant interpretation of "apparent mass", appears.

The essential point is that the equations will differ depending on the flow regime. A suspension of drops has different constitutive equations from a suspension of bubbles or a suspension of blood cells or a wavy stratified flow of two liquids. There is no universal formula to represent inertial coupling (though, perhaps, some general expression containing arbitrary coefficients might be adaptable to a variety of situations).

The authors should have qualified their derivation by restricting its application to flow regimes (I am not sure which they are) that can reasonably be described by their [2].

An additional problem arises in practice because the coupling may itself depend on the way in which an accelerating flow develops; moreover, it is usually when the velocity is changing rapidly that the inertial coupling is significant compared with the other terms in the equations of motion. If a bubble changes its shape, for example, its apparent mass also changes. An annular gas–liquid flow with a smooth interface has complex characteristics—a situation that is likely to be resolved by the development of interfacial waves that introduce inertial and frictional coupling. The constitutive equations, at least at the one-dimensional level, then contain additional and even changing terms and may require special methods of solution.

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